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Comparison of Coherent WDMA and Hybrid WDMA/CDMA for the Multiplexing of Optical Signals

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COMPARISON OF COHERENT WDMA AND HYBRID WDMA/CDMA FOR THE MULTIPLEXING OF OPTICAL SIGNALS

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ABSTRACT

In this paper, we provide an accurate analysis of the performance of coherent dense wavelength-division multiple-access (WDMA) schemes introduced for use in high-capacity optical networks. In our analysis, the effects of interference from other signals due to the frequency overlap caused by the instability of the carrier frequency of laser, or to mistakes in frequency coordination and assignment, are taken into account. Phase noise and thermal noise are also taken into consideration. Dense WDMA is then coupled with spread-spectrum direct-sequence modulation in order to mitigate the effect of interference from other signals. The performance of this hybrid of WDMA and code-division multiple-access (CDMA) scheme is also analyzed and compared to that of pure WDMA.

The average bit error probability of dense WDMA and WDMA/CDMA schemes is evaluated by integrating the characteristic function of other-user interference at the output of the matched optical filter. Gaussian approximation techniques are also employed. Time-synchronous and as asynchronous systems are analyzed in this context. Binary phase-shift-keying (BPSK) data modulation is considered. Our analysis quantifies accurately for first time the multiple-access capability of dense WDMA schemes and the advantages offered by employing hybrids of WDMA and CDMA.

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1. Introduction

This paper is motivated by the recent work of [1], where a new multiplexing scheme based on combination of dense wavelength-division multiple-access (WDMA) and code-division multiple-access (CDMA) schemes was introduced for use in high-capacity optical networks. According to this scheme, coherent optical techniques are employed to exploit the huge bandwidth (tens of thousands of GHz) of single-mode optical fibers through the use of dense WDMA. The carrier wavelengths employed by WDMA are placed relatively close to each other in order to maximize the number of distinct broadband channels (each may be 1 GHz or less) that can be accommodated.

Dense WDMA systems suffer from interference from other signals (termed other-user interference) due to the frequency overlap caused by the inherent instability of the carrier frequency of laser, or to mistakes in the frequency coordination and assignment. This interference can be mitigated with proper filtering, adequate separation between the laser frequency carriers, or the use of CDMA. In particular, spread-spectrum direct-sequence modulation was proposed in [1] as a means of mitigating interference from other signals due to such frequency overlap.

The work of [1] was preceded by that of [2] which introduced and analyzed random-carrier (RC) CDMA schemes, according to which the optical multiple-access system (assumed to place randomly the modulated carriers in the available optical band) is coupled with CDMA. For the purpose of simplifying the analysis, several assumptions were made in [2] about the noise environment (for example, no thermal noise or phase noise were included) and about the key parameters of the system (such as the spreading gain and the number of interfering users), which were assumed infinitely large, so that limiting theorems (e.g., the Central Limit Theorem or CLT) could be used. Furthermore, the accuracy of the approximations used was not justified for the range of the parameters of interest. Most of the assumptions of [2] were relaxed in our own work of [3], where an exact evaluation of the performance of the RC CDMA scheme was conducted without making any of the approximations and limiting assumptions made in [2].

More specifically, the assumptions made in [2] and relaxed in [3] were: (i) the number of interfering users was taken to be infinite; (ii) the number of chips per bit (spreading gain) of the spread-spectrum modulation was assumed to be very large so that the CLT could be used; (iii) the error probability was not directly evaluated and the outage probability was calculated in terms of the signal-to-noise ratio, which required the signals at the output of the optical matched filter to be Gaussian in order to be valid; (iv) the time delays of the various (possibly asynchronous) users did not enter in the evaluation of the performance; and (iv) the effects of the phase noise and

thermal noise on the system performance were not taken into account, which included not only the effects of phase noise on single-user performance, but also the effects of phase noise on the CDMA system performance caused by its effect on the interference terms due to other users.

Relaxing the above assumptions for the system of [2], and in particular assumptions (i) an (ii), was especially important for enabling the use of the techniques of [3] in evaluating the performance of WDMA and hybrid WDMA/CDMA schemes. Specifically, in the WDMA/CDMA applications the spreading gain is not very large, because the bandwidth is used more efficiently by the WDMA and only a relatively small part of it is dedicated to CDMA, in order to achieve desirable interference mitigation. Also, for pure WDMA schemes, the number of interfering users is not large, because only a small to moderate number of users occupying adjacent frequency bands cause interference.

The contribution of the current paper is twofold: First, it analyzes dense WDMA systems and evaluates their performance in terms of average error probability and the number of users supported by a specific bandwidth; specifically, it provides an accurate analysis of the performance of coherent WDMA schemes. In our analysis, the effects of interference from other signals due to frequency overlap are taken into account accurately and so are phase and thermal noise. Second, this paper evaluates the performance of hybrid WDMA/CDMA schemes and compares them to pure WDMA systems of the same bandwidth. Dense WDMA is then coupled with spread-spectrum direct-sequence modulation in order to mitigate the effect of interference from other signals.

The transmitter/receiver model of the hybrid WDMA/CDMA system is the one provided in [1]. The analysis is based on techniques used in [3]. The average bit error probability of a typical receiver for the dense WDMA and WDMA/CDMA schemes is evaluated by integrating the characteristic function of other-user interference at the output of the matched optical filter. Gaussian approximation techniques are also employed. Time-synchronous, as well as asynchronous systems, are analyzed in this context. Binary phase-shift-keying (BPSK) data modulation is considered.

2. Model

K high data rate users share a common optical channel in a multi-access fashion. These users are equally spaced in an optical bandwidth as big as $W = 10THz$ (see Fig. 1). Due to the frequency instabilities of lasers, each carrier frequency wanders around its designated frequency as much as $W_1 = 500GHz$. The transmitted optical signal is $S(t)$, which is a complex signal, is

$$S(t) = \sum_{m=1}^{K} \sqrt{P}\ b_m(t-\tau_m)a_m(t-\tau_m)e^{i[\omega_m(t-\tau_m)+\theta_m(t)]} \tag{2.1}$$

where associated parameters are as follows:

- P is the transmitted signal power of each user
- $b_m(t)$ is the data stream of the m-th user given by

$$b_m(t) = \sum_{n=-\infty}^{\infty} b_n^{(m)} p(t - nT)$$

  where $b_n^{(m)}$ denotes the n-th bit of the m-th user and $b_n^{(m)} \epsilon \{-1, 1\}$ for BPSK modulation. $p(t)$ is a pulse of unit amplitude in [0, T].

- $a_m(t)$ is the m-th addressing function or signature sequence stream for the hybrid WDMA/CDMA, that is,

$$a_m(t) = e^{i\phi_m(t)} = \sum_{n=-\infty}^{\infty} e^{i\phi_{mn}} h(t - nT_c)$$

  where $h(t)$ is a pulse of unit amplitude in $[0, T_c]$, $T_c = \frac{T}{N}$ the chip duration, and N the number of chips per bit. ϕ_{mn} is a phase taking values in $[-\pi, \pi]$. For pure WDMA, $a_m(t)$ is equal to the one in (2.1).

- ω_m is the carrier on which the m-th signal is sent. This value is uniformly distributed in a bandwidth of length W_1 (see Fig. 1).
- $\theta_m(t)$ is the phase noise associated with the m^{th} transmitter laser, which is a Brownian motion process with Lorentzian bandwidth β. The mean of this process is zero and the variance is $2\pi\beta t$.
- τ_m is the m-th time delay which is a uniform random variable distributed in [0, T]. For the synchronous system, this time delay is zero.

At the k-th receiver, the optical signal $S(t)$ is first despread by $a_k^*(t)$ (only for hybrid WDMA/CDMA), which is the complex conjugate of $a_k(t)$, and then homodyne-detected for the transmitted signal from user k (see Fig. 2). The output of the photodetector is

$$r(t) = \sqrt{P}\, b_k(t) e^{i\Delta\theta_k(t)} + \sum_m{}' \sqrt{P}\, b_m(t - \tau_m) e^{i[\omega'_m(t-\tau_m)+\phi_m(t-\tau_m)-\phi_k(t-\tau_m)+\Delta\theta_m(t)]} + n(t) \quad (2.2)$$

where

$$\sum_m{}' \triangleq \sum_{\substack{m=1 \\ m \neq k}}^{K}$$

$$\omega'_m \triangleq \omega_m - \omega_k$$

$$\Delta\theta_k(t) \triangleq \theta_k(t) - \theta_L(t)$$

$$\Delta\theta_m(t) \triangleq \theta_m(t) - \theta_L(t)$$

where $\theta_L(t)$ is the phase noise of the local laser and $n(t)$ the complex AWGN process with double-sided spectral density $\frac{N_0}{2}$. The term $\phi_m(t) - \phi_k(t)$ in (2.2) is not present for the WDMA case. The receiver used is a correlation receiver, which is optimum for the single user case with no phase noise (see Fig. 3). The real part of the output of the integrator is

$$Y = \left(b_0^{(k)} \cdot X + I + \eta\right) \sqrt{P} \tag{2.3}$$

where $I = \sum'_m I_m$ and I_m is the mth user interference. η is a zero mean Gaussian random variable of variance $N_0/2PT$ and X is

$$X = \frac{1}{T}\int_0^T \cos\left[\Delta\theta_k(t)\right] dt \tag{2.4}$$

The performance of this suboptimum receiver in the presense of phase noise and AWGN is obtained for WDMA and hybrid WDMA/CDMA by evaluating the characteristic function of multiuser interference.

3. WDMA

In this section, the performance of synchronous and asynchronous WDMA in terms of average probability of bit error is evaluated. The average probability of error is obtained as [.]

$$P_e = \overline{Q\left(X\sqrt{\frac{2PT}{N_0}}\right)} + \frac{1}{\pi}\int_0^\infty (1 - \Phi_I(u))\,\Phi_\eta(u)\overline{\frac{\sin(uX)}{u}}du \tag{3.1}$$

where

$$\Phi_\eta(u) = \exp(-\frac{N_0}{4PT}u^2)$$

and $\Phi_I(u)$ is the characteristic functions of I. The overlines denote the expectation with respect to the random variable X. The pdf of this random variable was obtained through the importance sampling simulation for different values of βT in [.]. The function $Q(\cdot)$ in (3.1) is related to the standard normal distribution as follows:

$$Q(\alpha) = \int_\alpha^\infty \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \cdot dx$$

(3.1) is derived under the assumption that I and X are independent. In the cases considered, this assumption is satisfied.

i) No phase noise

First we consider that no phase noise is present ($\beta T = 0$); then (3.1) is valid when $X = 1$. For the synchronous case, I_m is

$$I_m = \frac{b_0^{(m)}}{T} \int_0^T \cos(\omega'_m t)dt = b_0^{(m)} sinc(\omega'_m T). \tag{3.2}$$

The characteristic function of I is defined as

$$\Phi_I(u) = E\left[e^{iuI}\right] = E\left[e^{iu\sum'_m I_m}\right]. \tag{3.3}$$

Since $\{I_m\}$ are independent, it is reduced to

$$\Phi_I(u) = \prod_m{}' E\left[e^{iuI_m}\right] = \prod_m{}' E\left[e^{iub_0^{(m)} sinc(\omega'_m T)}\right]. \tag{3.4}$$

Since $b_0^{(m)}$ equally likely belongs to $\{-1, 1\}$, then

$$\Phi_I(u) = \prod_m{}' E\left[\cos(u\ sinc(\omega'_m T))\right]. \tag{3.5}$$

For the asynchronous case, I_m is

$$I_m = \frac{1}{T}\int_0^T b_m(t-\tau_m)\cos(\omega'_m t - \omega'_m \tau_m)dt \tag{3.6}$$

or

$$I_m = b_{-1}^{(m)} \frac{\tau_m}{T} sinc(\omega'_m \tau_m) + b_0^{(m)}(1 - \frac{\tau_m}{T}) sinc(\omega'_m (T - \tau_m)). \tag{3.7}$$

Following the same procedure as in the synchronous case, the characteristic function of I turns out to be

$$\Phi_I(u) = \prod_m{}' E\left\{\cos\left[u\frac{\tau_m}{T} sinc(\omega'_m \tau_m)\right]\cos\left[u(1-\frac{\tau_m}{T}) sinc(\omega'_m(T-\tau_m))\right]\right\}. \tag{3.8}$$

ii) Phase noise only for intended receiver

In this case, we assume that the phase noise is not present in other-user interference and that only the intended receiver suffers from phase noise. Average probability of bit error is obtained from (3.1), where X is no longer one. The characteristic function of the other-user interference $\Phi_I(u)$ is the same as in (3.5) and (3.8).

4) Hybrid WDMA/CDMA

The average probability of error is obtained as in (3.1). In order to find the multiuser characteristic function we need to know the other-user interference I_m, which is

$$I_m = \frac{1}{T}\int_0^T b_m(t-\tau_m)\cos\left[\omega_m' t - \omega_m'\tau_m + \phi_m(t-\tau_m) - \phi_k(t-\tau_m) + \Delta\theta_m(t)\right] dt. \tag{4.1}$$

For the synchronous case, τ_m is zero in (4.1). Let us partition this integral into N subintervals of length T_c such that $T_c N = T$ and assume that in each subinterval the noise term $\Delta\theta_m(t)$ remains constant. In other words,

$$\Delta\theta_m(t) = \Delta\theta_m(nT_c) \triangleq \theta_{mn} \qquad for\ (n-1)T_c < t \leq nT_c \tag{4.2}$$

where θ_{mn} is a zero-mean Gaussian random variable of variance $4\pi\beta nT_c$. Then, for the synchronous system, I_m is

$$I_m = \frac{b_0^{(m)}}{N} sinc\left(\frac{\omega_m' T_c}{2}\right)\sum_{n=1}^{N}\cos(X_{mn}) \tag{4.3}$$

where

$$X_{mn} =< \phi_{mn} - \phi_{kn} + \theta_{mn} + \omega_m'(n-1/2)T_c > . \tag{4.4}$$

For the asynchronous system, I_m is (see [3])

$$I_m = \frac{1}{N}\sum_{n=1}^{N}\left[e_{mn}^{+}\alpha_m^{+}\cos X_{mn}^{+} + e_{mn}^{-}\ \alpha_m^{-}\cos X_{mn}^{-}\right] \tag{4.5}$$

where

$$\alpha_m^{+} \triangleq \frac{\tau_m'}{T_c} sinc\left(\frac{\omega_m'\tau_m'}{2}\right) \tag{4.6}$$

$$\alpha_m^{-} \triangleq (1-\frac{\tau_m'}{T_c}) sinc\left[\frac{\omega_m'(T_c-\tau_m')}{2}\right] \tag{4.7}$$

$$e_{mn}^{+} \triangleq b_{-1}^{(m)}\cdot 1_{[1,\ell_m+1]}(n) + b_0^{(m)}\cdot 1_{[\ell_m+2,N]}(n) \tag{4.8}$$

$$e^{-}_{mn} \triangleq b^{(m)}_{-1} \cdot 1_{[1,\ell_m]}(n) + b^{(m)}_{0} \cdot 1_{[\ell_m+1,N]}(n) \tag{4.9}$$

$$1_{[i,j]}(n) \triangleq \begin{cases} 1 & i \leq n \leq j \\ 0 & \text{other} \end{cases} \tag{4.10}$$

$$X^{+}_{mn} \triangleq < \omega'_m(n-1)T_c + \frac{\omega'_m \tau'_m}{2} - \omega'_m \tau_m + \phi_{m(n-1)} - \phi kn + \theta_{mn} > \tag{4.11}$$

$$X^{-}_{mn} \triangleq < \omega'_m(n-1/2)T_c + \frac{\omega'_m \tau'_m}{2} - \omega'_m \tau_m + \phi_{mn} - \phi kn + \theta_{mn} > \tag{4.12}$$

and

$$\tau_m = \ell_m T_c + \tau'_m \tag{4.13}$$

where τ'_m is uniformly distributed in $[0, T_c]$ and ℓ_m is equally likely in $\{0, 1, \ldots, N-1\}$. In (4.4), (4.11), and (4.12), $< \cdot >$ represents $[\cdot]\ mod.2\pi$.

We adopt two approaches for obtaining the multiuser characteristic function. According to the first approach, it is assumed that the carrier frequencies are *uniformly* distributed in a bandwidth of $W1$. The phase signature sequences are uniformly distributed in the set of equally spaced phases $\{0, 2\pi/M, \ldots, (M-1)2\pi/M\}$, where M is the number of points in this set. This means that the phase signature sequence is modulated by an M-ary phase shift keying scheme with $M = 2$ being the most commonly used BPSK case.

According to the second approach, it is assumed that the phase signature sequences are *continuous* and *uniformly* distributed in $[-\pi, \pi]$. This approximates the case in which the number of levels M is large. In that case, the choice of the carrier frequencies distributions is arbitrary.

As will be shown below, the two approaches not only correspond to two different sets of assumptions that can be useful under various system conditions, but also are necessary for validating the use of the characteristic-function method and for the evaluation of the characteristic function of other-user interference.

1.Assumption of Uniform Carriers

We begin with the synchronous scheme. The following two lemmas are useful in our analysis (see [3] for proof).

- **Lemma 1**

  Let X be a random variable uniformly distributed in the bandwidth $[0, W]$. Let $Y = < X >$. Then, the distribution of Y approaches a uniform distribution in $[0, 2\pi]$ as $W \to \infty$.

- **Lemma 2**

Let the sequence $\{\phi_n\}_n$ be i.i.d. and uniform in $[0, 2\pi]$ and let the sequence $\{\lambda_n\}_n$ be arbitrary. Assume that these two sequences are independent for all n. Then the sequence $\{X_n\}_n$ defined as

$$X_n = <\phi_n + \lambda_n>$$

is i.i.d. and uniform in $[0, 2\pi]$.

Let X_{mn} in (4.4) be

$$X_{mn} = <\beta_{mn} + \gamma_{mn}> \tag{4.14}$$

where

$$\beta_{mn} = \omega'_m(n - 1/2)T_c \tag{4.15}$$

$$\gamma_{mn} = \phi_{mn} - \phi_{kn} + \theta_{mn}. \tag{4.16}$$

$\{\omega'_m\}$ is assumed to be independent and uniformly distributed in a bandwidth $W1$ as large as 500 GHz. Therefore, $\{\beta_{mn}\}$, for fixed n, are independent and uniformly distributed in a bandwidth of $(n-1/2)W1/(RN)$, where R is the data rate. For typical values of R and N, this value is still very large and according to the Lemma 1, we ought to consider $\{<\beta_{mn}>\}$ as i.i.d. random variables uniformly distributed in $[0, 2\pi]$. Consequently, Lemma 2 asserts that $\{X_{mn}\}$ in (4.14) are i.i.d. with respect to m and uniformly distributed in $[0, 2\pi]$. Therefore,

$$\Phi_I(u) = \prod_m^{I} E\left[e^{iu\frac{b_0^{(m)}}{N} sinc(\frac{\omega'_m T_c}{2}) \sum_{n=1}^{N} \cos(X_{mn})}\right]. \tag{4.17}$$

Conditioned on ω'_m in (4.17), it is easy to show that the sequence $\{\cos X_{mn}\}$ has zero-mean and zero correlation. Therefore, this sequence is a ρ- mixing one with $\rho(n) = 0$ (see [4] and [5]). We want to apply the CLT to the sequence $\{\cos X_{mn}\}_n$. To achieve that we state a theorem from [5].

- **Theorem**

  Let $\{X_n\}$ be a second order stationary, centered, and ρ-mixing sequence; let $\sigma_n^2 \to \infty$ and $\sum_i \rho(2^i) < \infty$. Then $\{s_n/\sigma_n\}$ satisfies the CLT, where $s_n = \sum_{i=1}^{n} X_i$ and $\sigma_n^2 = Var(s_n)$.

The variance of the sequence $\{\cos X_{mn}\}_n$ turns out to be

$$\sigma_{mn}^2 = \begin{cases} \frac{1}{2} + \frac{1}{2}\cos[\omega'_m(2n-1)T_c]E\{\cos(2\theta_{mn})\}, & M = 2 \\ \frac{1}{2}, & M > 2 \end{cases} \tag{4.18}$$

Since $2\theta_{mn}$ is a zero-mean Gaussian random variable of variance $16\pi\beta nT_C$, it is easy to show that the expectation in (4.18) is $\exp(-8\pi\beta nT_c)$. Hence,

$$\sigma_{mn}^2 = \begin{cases} \frac{1}{2} + \frac{1}{2}\cos[\omega'_m(2n-1)T_c]\exp(-8\pi\beta nT_c), & M = 2 \\ \frac{1}{2}, & M > 2 \end{cases} \tag{4.19}$$

Apparently, all conditions of the theorem are satisfied for $M > 2$. But, for $M = 2$, the sequence is not second-order stationary. The variance of the term $\frac{1}{\sqrt{N}} \sum_{n=1}^{N} \cos(X_{mn})$ is

$$\sigma_m^2 = \begin{cases} \frac{1}{2} + \frac{1}{2N}\Sigma_{n=1}^{N} \cos[\omega'_m(2n-1)T_c]\exp(-8\pi\beta nT_c), & M = 2 \\ \frac{1}{2}, & M > 2 \end{cases} \tag{4.20}$$

It is easy to show that

$$\lim_{N\to\infty} \frac{1}{N}\Sigma_{n=1}^{N} \cos[\omega'_m(2n-1)T_c]\exp(-8\pi\beta nT_c) = \int_0^1 \cos(2\omega'_m T)\exp(-8\pi\beta Tt)dt$$

which is easy to compute. Therefore, as $N \to \infty$, (4.20) converges to

$$\sigma_m^2 = \begin{cases} \frac{1}{2} + \frac{1}{2(a^2+b_m^2)}\left[(b_m \sin(b_m) - a\cos(b_m))\exp(-a) + a\right], & M = 2 \\ \frac{1}{2}, & M > 2 \end{cases} \tag{4.21}$$

where $a = 8\pi\beta T$ and $b_m = 2\omega'_m T$. This limit is positive (non zero) for almost all values of ω'_m. Despite the fact that, for $M = 2$, the condition of stationarity required by the theorem is not satisfied, we have shown that, for almost any given ω'_m, the sequence $\{\frac{1}{N}\sum_{n=1}^{N}\sigma_{mn}^2\}$ converges to a positive limit. This also satisfies the condition $\sum_{n=1}^{N}\sigma_{mn}^2 \to \infty$, as $N \to \infty$, stated in the theorem. We may thus proceed under the assumption that the CLT holds. Then, for large N, the term $\frac{1}{\sqrt{N}} \sum_{n=1}^{N} \cos(X_{mn})$ in (4.17) can be replaced by ζ_m , where ζ_m is a Gaussian random variable of zero mean and variance σ_m^2. Conditioned on $b_0^{(m)}$ and ω'_m, (4.17) becomes

$$\Phi_I(u) = \prod_m^{I} \left[E_{b_0^{(m)},\omega'_m} \left\{ E\left[e^{iu\frac{b_0^{(m)}}{\sqrt{N}} sinc(\frac{\omega'_m T_c}{2})\zeta_m} \right] \right\} \right]. \tag{4.22}$$

The inner expectation in (4.22) is the characteristic function of a Gaussian random variable. Therefore,

$$\Phi_I(u) = \prod_m^{I} \left[E\left\{ e^{-u^2\frac{b_0^{(m)2}}{2N}\left(sinc(\frac{\omega'_m T_c}{2})\right)^2 \sigma_m^2} \right\} \right] \tag{4.23}$$

where the expectation is with respect to $b_0^{(m)}$ and ω'_m. For BPSK, $b_0^{(m)} \epsilon \{-1, 1\}$ and consequently, we obtain

$$\Phi_I(u) = \prod_m^{I} \left[\exp\left\{ -\frac{u^2}{2N}\left(sinc(\frac{\omega'_m T_c}{2})\right)^2 \sigma_m^2 \right\} \right]. \tag{4.24}$$

The derivation for the asynchronous counterpart of (4.24) requires more work, although the same argument applies. The final result, which is mostly obtained in [3] for the random-carrier case, is

$$\Phi_I(u) = \prod_m^{I} \left[\frac{1}{N} \sum_{n=1}^{N} \Phi(u; m, n) \right] \tag{4.25}$$

where

$$\Phi(u; m, n) = E_{\tau'_m} E_{\omega'_m} \left[e^{\frac{-u^2}{2N} \sigma_\eta^2} \right] \tag{4.26}$$

$$\sigma_\eta^2 = [(\alpha_m^+)^2 + (\alpha_m^-)^2] \sigma_m^2 \tag{4.27}$$

$$\sigma_m^2 = \begin{cases} \frac{1}{2} + \frac{1}{2(a^2+b_m^2)} \left[\left(b_m \sin(b_m(1-\frac{n}{N}) - \omega'_m \tau'_m) - a\cos(b_m(1-\frac{n}{N}) - \omega'_m \tau'_m) \right) \exp(-a) \right. & \\ \left. + a\cos(\frac{n}{N} b_m + \omega'_m \tau'_m) + b_m \sin(\frac{n}{N} b_m + \omega'_m \tau'_m) \right] & M = 2 \\ \frac{1}{2} & M > 2 \end{cases} \tag{4.28}$$

2.<u>Assumption of Uniform Phase Signature Sequence</u>

For the case in which the signature sequence phases are uniformly distributed in a set of equally spaced discrete levels, if the number of levels in the set is reasonably large, this discrete uniform distribution is approximated with a continuous uniform phase in $[0, 2\pi]$. Let us express the characteristic function of the interference I as

$$\Phi_I(u) = E_{\overline{\omega'}} \; E_{\overline{b_0}} \; E_{\overline{X}} \left[e^{iuI} \right] \tag{4.29}$$

where I_m is given by (4.3) for the synchronous case. $E_{\overline{X}}$ is the expectation with respect to the $N(K-1)$ dimensional vector in $\{X_{mn}\}$. $E_{\overline{b_0}}$ is the expectation with respect to the $K-1$ dimensional vector in $\{b_0^{(m)}\}$. Finally, $E_{\overline{\omega'}}$ is the expectation with respect to the $K-1$ dimensional vector $\overline{\omega'} = (\omega'_1, \ldots, \omega'_K)$. Since $\{\phi_{mn}\}$ in (4.4) are i.i.d. and uniformly distributed in $[0, 2\pi]$ for all m and n , $\{X_{mn}\}$ are also i.i.d. and uniform in $[0, 2\pi]$, for all m and n, as shown in Lemma 2. Therefore,

$$\Phi_I(u) = \prod_m^{I} \; E_{\omega'_m} \; \left\{ E_{b_0^{(m)}} \left\{ \prod_{n=1}^{N} \; E_{X_{mn}} \left[e^{i \frac{u}{N} b_0^{(m)} \cdot \mathrm{sin\,c} \left(\frac{\omega'_m T_c}{2} \right) \cos(X_{mn})} \right] \right\} \right\} . \tag{4.30}$$

We use the identity

$$E_{X_{mn}} \left[e^{iu\cos(X_{mn})} \right] = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{iu\cos x} \cdot dx =$$

$$= \frac{2}{\pi} \int_0^{\pi/2} \cos(u \cos x) \, dx = J_0(u) \tag{4.31}$$

where $J_0(\cdot)$ is the Bessel function of the first kind. By using (4.31), the inner expectation in (4.30) becomes $J_0\left(b_0^{(m)} \cdot \frac{u}{N} \cdot \sin c\left(\frac{\omega'_m T_c}{2}\right)\right)$, which is independent of n. Also, by using the fact that $J_0(\alpha) = J_0(-\alpha)$, we obtain that

$$E_{b_0^{(m)}}\left\{\prod_{n=1}^{N} E_{X_{mn}}[\cdot]\right\} = \left[J_0\left(\frac{u}{N} sinc\left(\frac{\omega'_m T_c}{2}\right)\right)\right]^N. \tag{4.32}$$

Finally,

$$\Phi_I(u) = \prod_m' \left\{\overline{\left[J_0\left(\frac{u}{N} sinc\left(\frac{\omega'_m T_c}{2}\right)\right)\right]^N}\right\} \tag{4.33}$$

where the overline is the expectation with respect to the ω'_m.

Similarly, the asynchronous counterpart of (4.33) is [3]

$$\Phi_I(u) = \prod_m' \left\{\overline{\left[J_0\left(\frac{u}{N} \cdot \frac{\tau'_m}{T_c} sinc\left[\frac{\omega'_m \tau'_m}{2}\right]\right) J_0\left(\frac{u}{N}\left(1 - \frac{\tau'_m}{T_c}\right) sinc\left[\frac{\omega'_m (T_c - \tau'_m)}{2}\right]\right)\right]^N}\right\} \tag{4.34}$$

where the expectation in (4.34) is with respect to ω'_m and τ'_m.

5. Conclusion

In Fig. 4, the single user BER which is a decreasing function of SNR is illustrated. The considerable degradation of performance due to phase noise is clear. Fig. 5 depicts only the multiuser portion of BER (P_{em}) for WDMA, where the case of $K = 101$ users of rate 10 Mbps is considered. The synchronous and asynchronous systems differ slightly. This and the results in [3] assert that synchronization does not provide any substantial enhancement in performance and the deterioration of P_{em} due to phase noise is negligible. Fig. 6 compares the two methods in obtaining hybrid performance. The results are very close, which indicates the validity of the CLT in the first method. Also, notice that, for $M > 2$ and $M = 2$, the results are almost identical. Fig. 7 compares WDMA and hybrid WDMA/CDMA for $K = 2001$ users of rate 10 Mbps. At a higher SNR, which is the region of practical interest, the hybrid scheme performs considerably better. Finally, Fig. 8 compares these two schemes in terms of the total BER for $K = 2001$ users. At a lower SNR, the single user portion of error is dominating and thus both schemes are the same; however, at a higher SNR, the superiority of the hybrid scheme is apparent. This result, in addition to the protection mechanism of the CDMA in case of frequency overlaps, establishes the hybrid WDMA/CDMA as a viable alternative to pure WDMA.

References

1. G. Vannucci. "Combining Frequency-Division and Code-Division Multiplexing in a High-Capacity Optical Network." *IEEE Network*, March 1989, 21-30.

2. G. J. Foschini and G. Vannucci. "Using Spread Spectrum in a High-Capacity Fiber-Optic Local Network." *Journal of Lightwave Technology*, Vol. 6, March 1988, pp. 370-379.

3. B. Ghaffari and E. Geraniotis. "Analysis of Coherent Random-Carrier Code-Division Multiple-Access for High-Capacity Optical Networks." Submitted to the *IEEE Transactions on Communications*, 1990, under review.

[4] R. C. Bradley, "The Basic Properties of Strong Mixing Conditions," in *Dependence in Probability and Statistics*, Birkhauser, 1985.

[5] M. Peligrad, "Recent Advances in the Central Limit Theorem and its Weak Invariance Principle for Mixing Sequences of Random Variables," in *Dependence in Probability and Statistics*, Birkhauser, 1985.

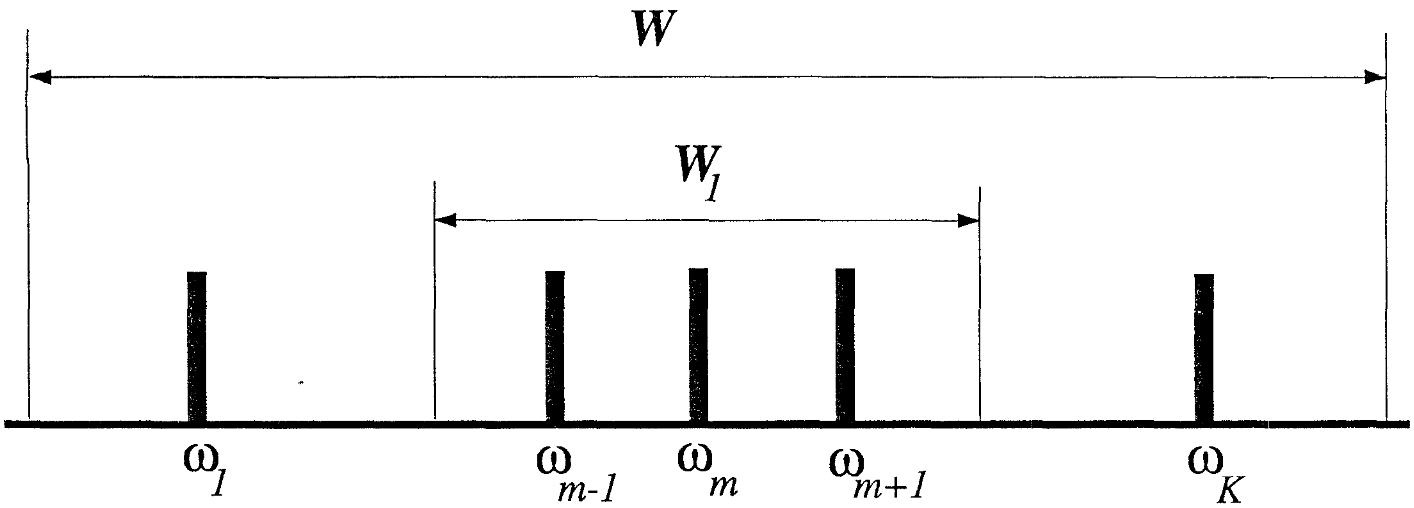

Fig. 1 Allocation of carrier frequencies on the optical band

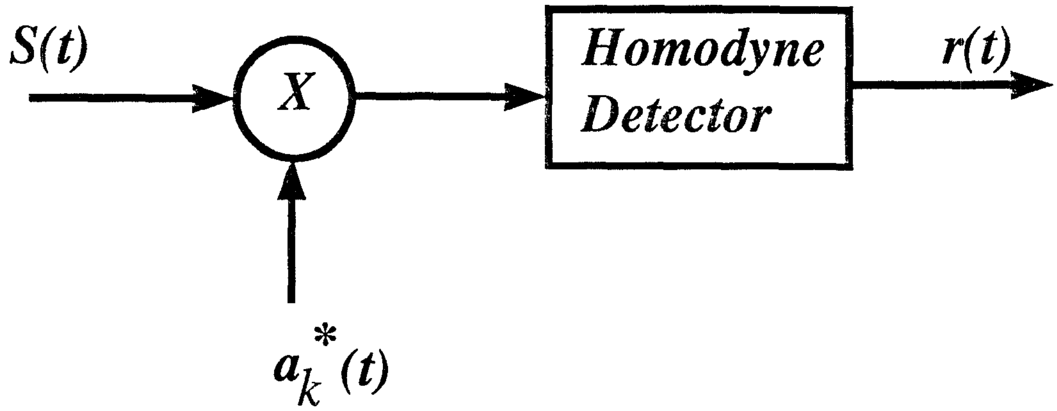

Fig.2 Coherent detection of the optical signal $S(t)$

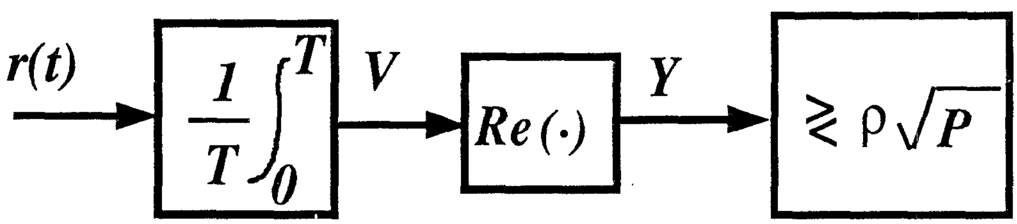

Fig.3 Detection of the electrical signal $r(t)$

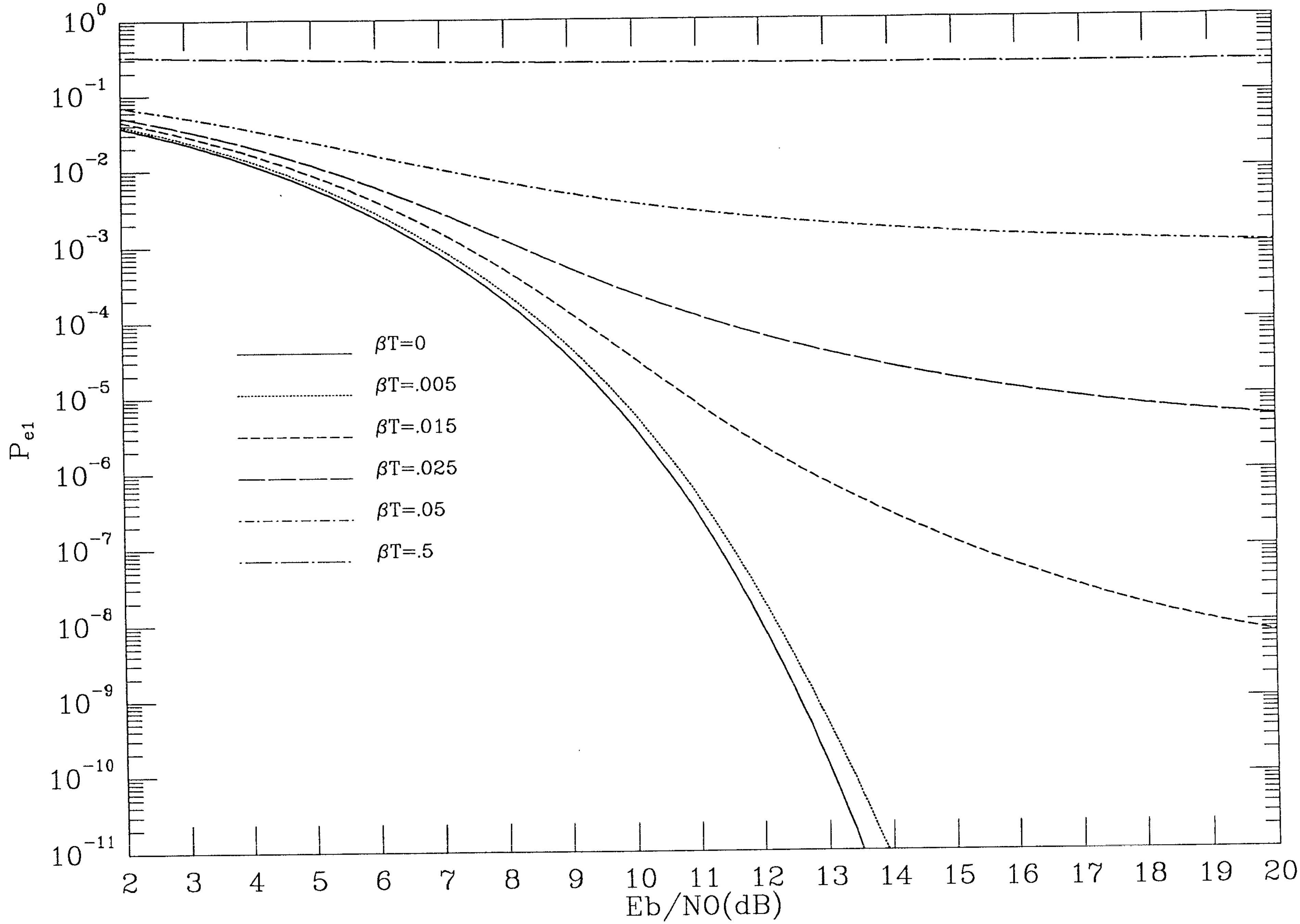

Fig. 4. Performance of the single user system (BPSK)

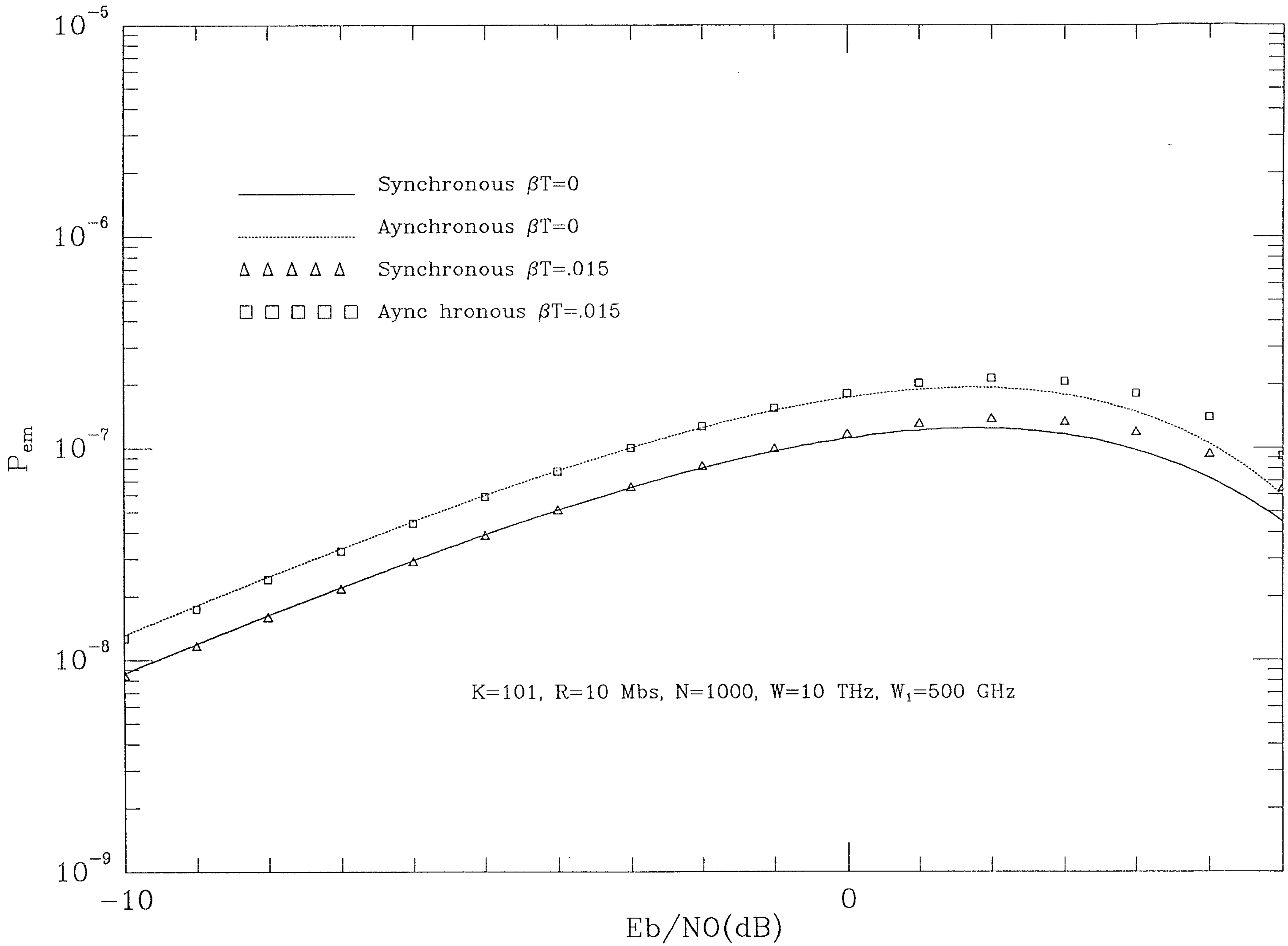

Fig. 5. Synchronous and Asynchronous WDMA

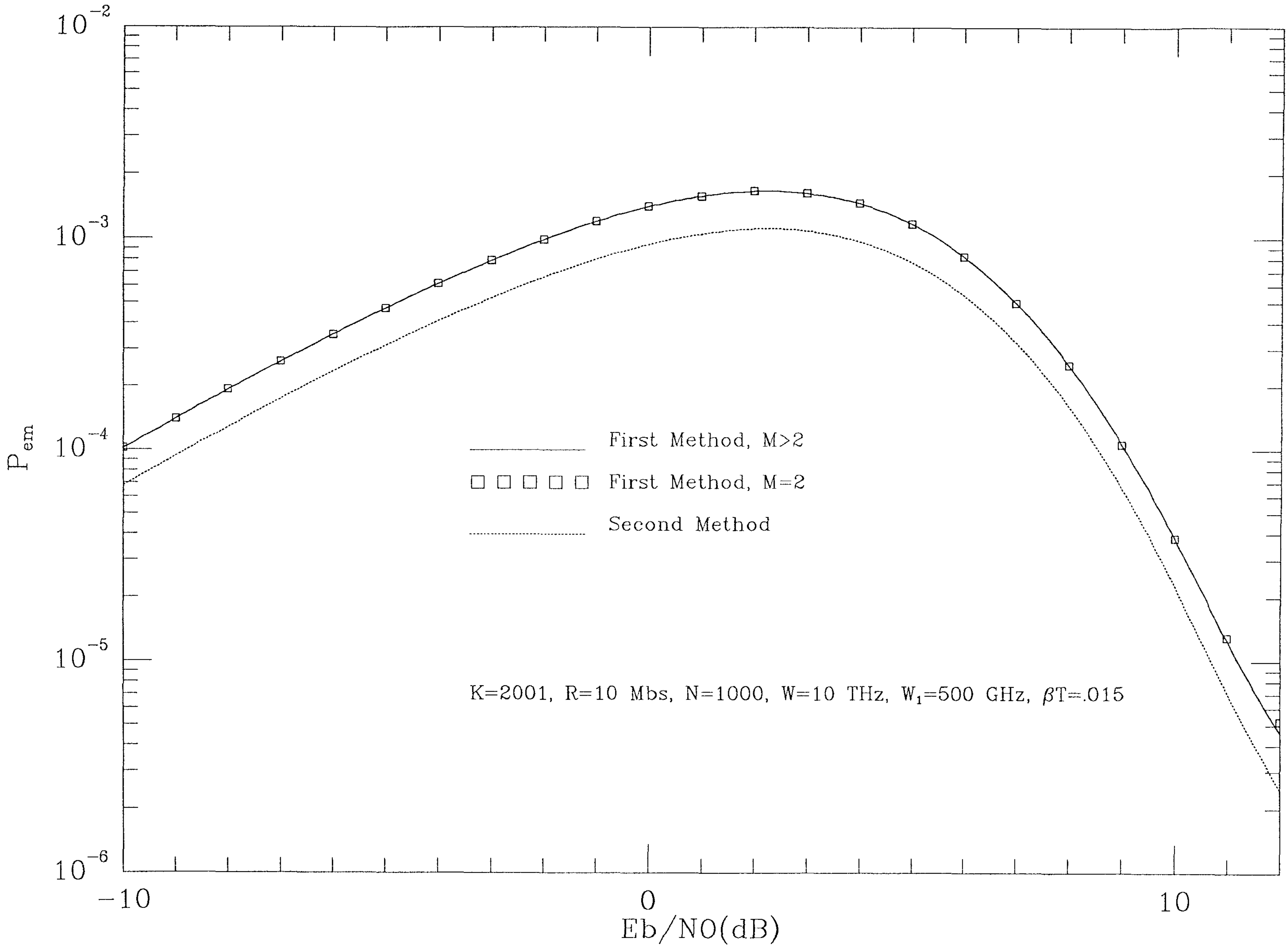

Fig. 6. Comparison of two methods in hybrid WDMA/CDMA (synchronous)

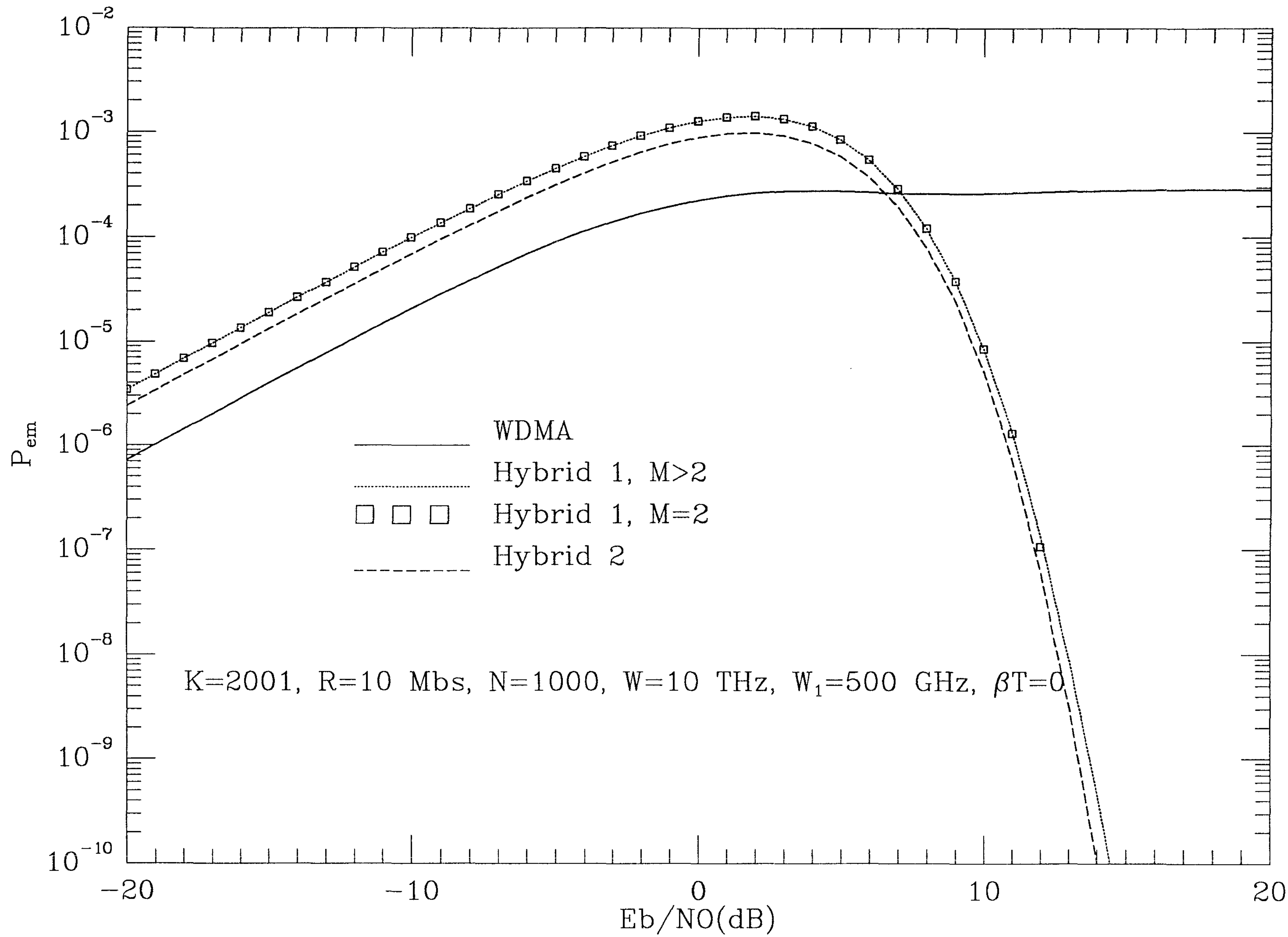

Fig. 7. Comparison of WDMA and hybrid WDMA/CDMA (synchronous)

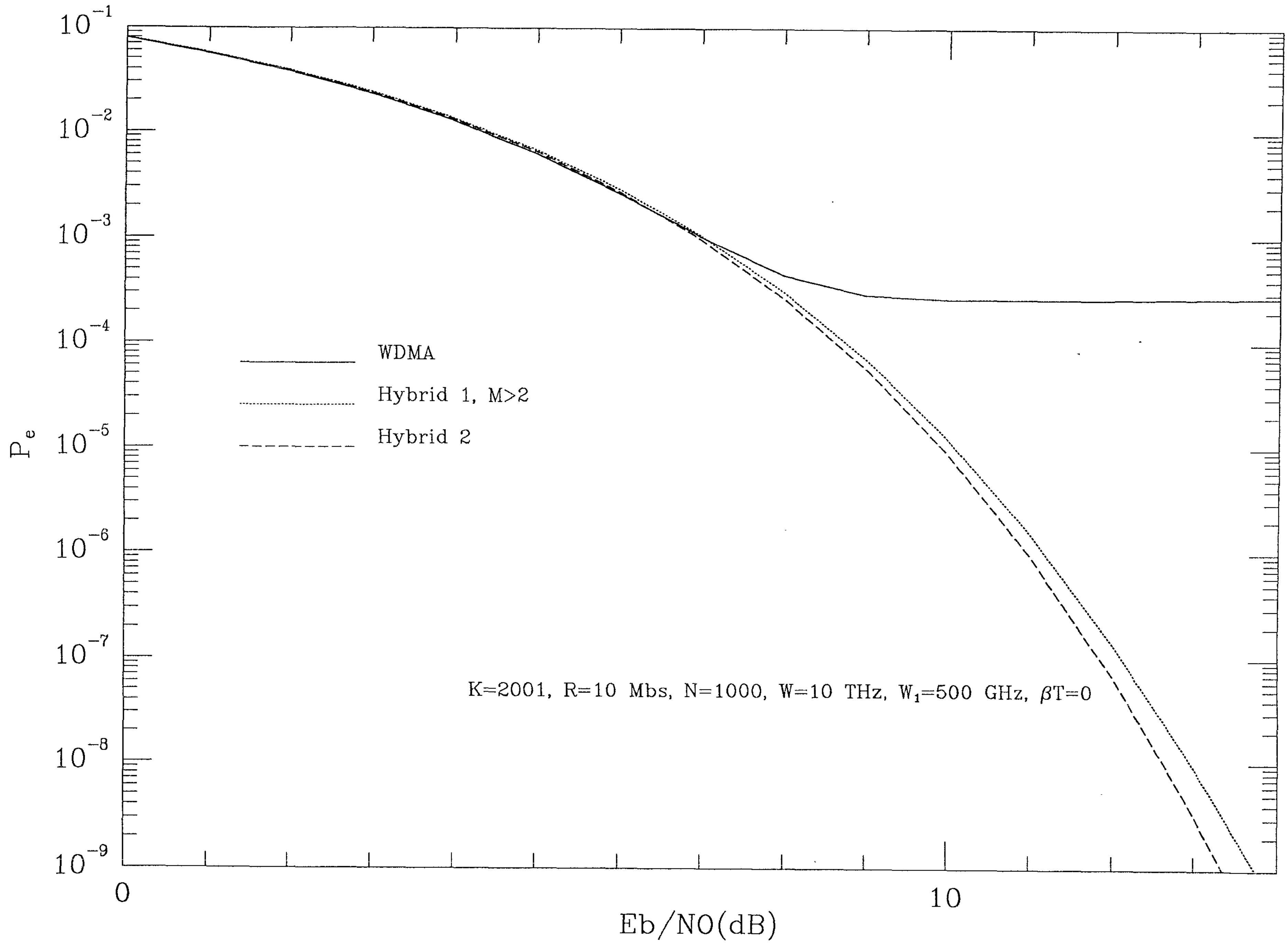

Fig. 8. Comparison of WDMA and hybrid WDMA/CDMA (synchronous)